Math 10/11 Honors Review: Trigonometry

1 Given a circle with radius 6 cm and length of arc 12 cm, determine the sector angle to the nearest degree.

a) 2° **b)** 29° **c)** 108° **d)** 115°

2 If $\csc x = \frac{-2}{\sqrt{3}}$ and $\tan x < 0$, determine $\cos x$.

a) $\frac{-1}{2}$ **b)** $\frac{-\sqrt{3}}{2}$ **c)** $\frac{1}{2}$ **d)** $\frac{\sqrt{3}}{2}$

3 Given a circle with radius 6 cm and length of arc 12 cm, determine the sector angle to the nearest degree.

a) 2° **b)** 29° **c)** 108° **d)** 115°

3b If $(2, -3)$ is on the terminal side of standard position angle θ, what is the value of $\sec\theta$?

a) $\frac{-\sqrt{13}}{2}$ **b)** $\frac{\sqrt{13}}{2}$ **c)** $\frac{-\sqrt{13}}{3}$ **d)** $\frac{\sqrt{13}}{3}$

4 Determine the smallest positive angle θ, in radians, such that $\csc\theta = -\sqrt{2}$

a) $\frac{\pi}{4}$ **b)** $\frac{3\pi}{4}$ **c)** $\frac{5\pi}{4}$ **d)** $\frac{7\pi}{4}$

5 Convert 10 radians to a degree value between 0° and 360°.

a) 33° **b)** 148° **c)** 213° **d)** 303°

6 Determine the exact value of $\csc\frac{5\pi}{3}$.

a) $\frac{-\sqrt{3}}{3}$ **b)** $\frac{-2\sqrt{3}}{3}$ **c)** $\frac{\sqrt{3}}{3}$ **d)** $\frac{2\sqrt{3}}{3}$

7 Simplify: $\frac{\sin 2x}{1+\cos 2x}$.

a) $\tan x$ **b)** $\csc x$ **c)** $\sec x$ **d)** $\cot x$

8 Which expression is equivalent to: $\frac{\sin 6x}{\sin 3x}$?

a) $\sin 2x$ **b)** $2\cos 3x$ **c)** $2\sin 3x$ **d)** $3\cos 3x$

9 Simplify: $\cos\frac{x}{3}\sin\frac{x}{3}$.

a) $\frac{1}{2}\sin\frac{2}{3}x$ **b)** $2\sin\frac{x}{3}$ **c)** $2\sin\frac{2}{3}x$ **d)** $\tan\frac{x}{3}$

10 Simplify: $\sin^2 x + \cos^2 x + \cot^2 x$.

a) $\csc^2 x$ **b)** $\sec^2 x$ **c)** $\tan^2 x$ **d)** $2\sin^2 x$

11 Simplify: $-1-\cos^2 x+\sin^2 x$.

a) -2 **b)** 0 **c)** 2 **d)** $-2\cos^2 x$

12 Simplify: $\dfrac{\cos^3 A-\cos A}{\sin^3 A}$.

a) $\cot A$ **b)** $-\cot A$ **c)** $\tan A$ **d)** $-\tan A$

13 . What is the value of $\tan B$ if $\cos B=\dfrac{-3}{5}$ and $\pi \le B<\dfrac{3\pi}{2}$?

a) $\dfrac{3}{4}$ **b)** $\dfrac{-3}{4}$ **c)** $\dfrac{4}{3}$ **d)** $\dfrac{-4}{3}$

14 Simplify: $\dfrac{2\tan x}{1+\tan^2 x}$.

a) $\sin 2x$ **b)** $2\sin x$ **c)** $\cos 2x$ **d)** $2\cos x$

15 What is/are the restriction(s) for the expression: $\dfrac{\csc x}{\cos x}$?

a) $\csc x \neq 0$ **b)** $\cos x \neq 0$ **c)** $\sin x \neq 0$ **d)** $\sin x \neq 0,\ \cos x \neq 0$

16 Simplify: $\dfrac{\sec A-\cos A}{\tan A}$

a) $\sin A$ **b)** $\cos A$ **c)** $\sec A$ **d)** $\csc A$

17 Simplify: $\cos\dfrac{x}{4}\cos\dfrac{x}{3}-\sin\dfrac{x}{4}\sin\dfrac{x}{3}$.

a) $\sin\dfrac{x}{12}$ **b)** $\sin\dfrac{7x}{12}$ **c)** $\cos\dfrac{x}{12}$ **d)** $\cos\dfrac{7x}{12}$

18 How many solutions does $\cos^2 x=\dfrac{1}{a}$, $a>1$, $0\le x<2\pi$

a) 1 **b)** 2 **c)** 3 **d)** 4

19 How many solutions does the following equation have over the interval, $0^\circ \le \theta < 360^\circ$.

$(a\sin\theta-a)(\tan^3 x-b)=0$

a) 2 **b)** 3 **c)** 4 **d)** 5

20 Simplify: $\dfrac{1+\sec x}{\tan x+\sin x}$

a) $\csc x$ **b)** $\sec x$ **c)** $\tan x$ **d)** $\cot x$

21	Simplify: $\dfrac{\cos x}{1-\sin x}-\dfrac{1}{\cos x}$ **a)** $\tan x$ **b)** $\cot x$ **c)** $\sec x$ **d)** $\csc x$
22	Simplify: $\dfrac{\csc^2 x-1}{\sec^2 x-1}$. **a)** $2\tan^2 x$ **b)** $2\cot^2 x$ **c)** $\tan^4 x$ **d)** $\cot^4 x$
23	Simplify: $\cos\left(\dfrac{\pi}{6}+x\right)\cdot\cos\left(\dfrac{\pi}{6}-x\right)-\sin\left(\dfrac{\pi}{6}+x\right)\cdot\sin\left(\dfrac{\pi}{6}-x\right)$ **a)** 0 **b)** $\dfrac{1}{2}$ **c)** $\dfrac{\sqrt{3}}{2}$ **d)** $\dfrac{2\pi}{3}$
24	Simplify: $\dfrac{\sin A}{1+\cos A}-\dfrac{1-\cos A}{\sin A}$ **a)** 0 **b)** $2\csc A$ **c)** $2\sec A$ **d)** $\dfrac{2\sin A}{1+\cos A}$
25	Simplify $(\sin x-\cos x)^2-(\sin x+\cos x)^2$. **a)** 0 **b)** $-\sin 2x$ **c)** $\sin 2x$ **d)** $-2\sin 2x$

Prove the following identities:

a) $\dfrac{\sec x}{1-\cos x}=\dfrac{\sec x+1}{\sin^2 x}$

b) $\dfrac{\sin\theta+\cos\theta\cdot\cot\theta}{\cos\theta\csc\theta}=\sec\theta$

c) $\dfrac{1+\sec\theta}{\sin\theta+\tan\theta}=\csc\theta$

d) $\dfrac{\sec x}{1-\sin x}=\dfrac{1+\sin x}{\cos^3 x}$

e) $\cos^2 x=\dfrac{1-2\sin^2 x}{1-\tan^2 x}$

f) $\tan x=\csc 2x-\cot 2x$

g) $\dfrac{1-\cos\theta}{\sin\theta}=\dfrac{1}{\csc\theta+\cot\theta}$

h) $\dfrac{\cos 2\theta}{\sin\theta}+\dfrac{\sin 2\theta}{\cos\theta}=\csc\theta$

i) $\dfrac{\sin^2 x-\tan x}{\cos^2 x-\cot x}=\tan^2 x$

j) $\cos 2x=\dfrac{\cot x-\tan x}{\cot x+\tan x}$

o) $\tan 2x=\dfrac{2}{\cot x-\tan x}$

k) $\csc x-\dfrac{\sin x}{1+\cos x}=\cot x$

l) $\cot x-\tan x=\dfrac{4\cos^2 x-2}{\sin 2x}$

q) $\dfrac{\cos x}{\csc x+1}+\dfrac{\cos x}{\csc x-1}=2\tan x$

m) $\dfrac{1-\sin x}{1+\sin x}=(\sec x-\tan x)^2$

n) $\dfrac{\sin 4x-\sin 2x}{\cos 4x+\cos 2x}=\tan x$

q) $\dfrac{\cos x}{\csc x+1}+\dfrac{\cos x}{\csc x-1}=2\tan x$

a)

$$\frac{\sec x}{1-\cos x}=\frac{\sec x+1}{\sin^2 x}$$

$$\frac{\sec x}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x}=$$

$$\frac{\sec x+1}{1-\cos^2 x}=$$

$$\frac{\sec x+1}{\sin^2 x}=$$

b)

$$\frac{\sin\theta+\cos\theta\cdot\cot\theta}{\cos\theta\csc\theta}=\sec\theta$$

$$\frac{\sin\theta+\cos\theta\cdot\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta}{\sin\theta}}=\frac{1}{\cos\theta}$$

$$\frac{\sin^2\theta+\cos^2\theta}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}=$$

$$\frac{1}{\cos\theta}=$$

c)

$$\frac{1+\sec\theta}{\sin\theta+\tan\theta}=\csc\theta$$

$$\frac{1+\dfrac{1}{\cos\theta}}{\sin\theta+\dfrac{\sin\theta}{\cos\theta}}=$$

$$\frac{\dfrac{\cos\theta+1}{\cos\theta}}{\dfrac{\sin\theta\cdot\cos\theta+\sin\theta}{\cos\theta}}=$$

$$\frac{\cos\theta+1}{\sin\theta(\cos\theta+1)}=$$

$$\csc\theta=$$

d)

$$\frac{\sec x}{1-\sin x}=\frac{1+\sin x}{\cos^3 x}$$

$$\frac{\sec x(1+\sin x)}{(1-\sin x)(1+\sin x)}=$$

$$\frac{\sec x(1+\sin x)}{1-\sin^2 x}=$$

$$\frac{\sec x(1+\sin x)}{\cos^2 x}=$$

$$\frac{1+\sin x}{\cos^3 x}=$$

e)

$$\cos^2 x=\frac{1-2\sin^2 x}{1-\tan^2 x}$$

$$=\frac{1-2\sin^2 x}{1-\dfrac{\sin^2 x}{\cos^2 x}}$$

$$=\frac{1-2\sin^2 x}{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}}$$

$$=\frac{\cos^2 x(1-2\sin^2 x)}{\cos^2 x-\sin^2 x}$$

$$=\frac{\cos^2 x(1-2\sin^2 x)}{1-\sin^2 x-\sin^2 x}$$

$$=\frac{\cos^2 x(1-2\sin^2 x)}{1-2\sin^2 x}$$

$$=\cos^2 x$$

f)

$$\tan x=\csc 2x-\cot 2x$$

$$=\frac{1}{\sin 2x}-\frac{\cos 2x}{\sin 2x}$$

$$=\frac{1-\cos 2x}{\sin 2x}$$

$$=\frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$$

$$=\frac{2\sin^2 x}{2\sin x\cos x}$$

$$=\frac{\sin x}{\cos x}$$

$$=\tan x$$

g)

$$\frac{1-\cos\theta}{\sin\theta}=\frac{1}{\csc\theta+\cot\theta}$$

$$\frac{1-\cos\theta}{\sin\theta}\cdot\frac{(1+\cos\theta)}{(1+\cos\theta)}=\frac{1}{\dfrac{1}{\sin\theta}+\dfrac{\cos\theta}{\sin\theta}}$$

$$\frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}=\frac{1}{\dfrac{1+\cos\theta}{\sin\theta}}$$

$$\frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}=\frac{\sin\theta}{1+\cos\theta}$$

$$\frac{\sin\theta}{1+\cos\theta}=$$

h)

$$\frac{\cos 2\theta}{\sin\theta}+\frac{\sin 2\theta}{\cos\theta}=\csc\theta$$

$$\frac{\cos 2\theta\cdot\cos\theta+\sin 2\theta\cdot\sin\theta}{\sin\theta\cos\theta}=$$

$$\frac{\cos(2\theta-\theta)}{\sin\theta\cos\theta}=$$

$$\frac{\cos\theta}{\sin\theta\cos\theta}=$$

$$\frac{1}{\sin\theta}=$$

$$\csc\theta=$$

i)

$$\frac{\sin^2 x-\tan x}{\cos^2 x-\cot x}=\tan^2 x$$

$$\frac{\sin^2 x-\dfrac{\sin x}{\cos x}}{\cos^2 x-\dfrac{\cos x}{\sin x}}=\frac{\sin^2 x}{\cos^2 x}$$

$$\frac{\dfrac{\sin^2 x\cos x-\sin x}{\cos x}}{\dfrac{\sin x\cos^2 x-\cos x}{\sin x}}=$$

$$\frac{\sin x(\sin x\cos x-1)}{\cos x(\sin x\cos x-1)}\cdot\frac{\sin x}{\cos x}=$$

$$\frac{\sin^2 x}{\cos^2 x}=$$

j)

$$\cos 2x=\frac{\cot x-\tan x}{\cot x+\tan x}$$

$$=\frac{\dfrac{\cos x}{\sin x}-\dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\sin x}+\dfrac{\sin x}{\cos x}}$$

$$=\frac{\dfrac{\cos^2 x-\sin^2 x}{\sin x\cos x}}{\dfrac{\sin^2 x+\cos^2 x}{\sin x\cos x}}$$

$$=\frac{\cos^2 x-\sin^2 x}{\sin^2 x+\cos^2 x}$$

$$=\cos 2x$$

k)

$$\csc x-\frac{\sin x}{1+\cos x}=\cot x$$

$$\frac{1}{\sin x}-\frac{\sin x}{1+\cos x}=\frac{\cos x}{\sin x}$$

$$\frac{1+\cos x-\sin^2 x}{\sin x(1+\cos x)}=$$

$$\frac{\cos x+\cos^2 x}{\sin x(1+\cos x)}=$$

$$\frac{\cos x(1+\cos x)}{\sin x(1+\cos x)}=$$

$$\frac{\cos x}{\sin x}=$$

l)

$$\cot x-\tan x=\frac{4\cos^2 x-2}{\sin 2x}$$

$$\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}=\frac{2(2\cos^2 x-1)}{\sin 2x}$$

$$\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}=\frac{2\cos 2x}{\sin 2x}$$

$$\frac{\cos 2x}{\frac{1}{2}\sin 2x}=$$

$$\frac{2\cos 2x}{\sin 2x}=$$

m)

$$\frac{1-\sin x}{1+\sin x}=(\sec x-\tan x)^2$$

$$=\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)^2$$

$$=\frac{(1-\sin x)^2}{\cos^2 x}$$

$$=\frac{(1-\sin x)^2}{1-\sin^2 x}$$

$$=\frac{(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$$

$$=\frac{1-\sin x}{1+\sin x}$$

n)

$$\frac{\sin 4x-\sin 2x}{\cos 4x+\cos 2x}=\tan x$$

$$\frac{2\sin 2x\cos 2x-\sin 2x}{2\cos^2 2x-1+\cos 2x}=\frac{\sin x}{\cos x}$$

$$\frac{\sin 2x(2\cos 2x-1)}{(2\cos 2x-1)(\cos 2x+1)}=$$

$$\frac{2\sin x\cos x}{(2\cos^2 x-1)+1}=$$

$$\frac{2\sin x\cos x}{2\cos^2 x}=$$

$$\frac{\sin x}{\cos x}=$$

o)

$$\tan 2x=\frac{2}{\cot x-\tan x}$$

$$\frac{\sin 2x}{\cos 2x}=\frac{2}{\dfrac{\cos x}{\sin x}-\dfrac{\sin x}{\cos x}}$$

$$=\frac{2}{\dfrac{\cos^2 x-\sin^2 x}{\sin x\cos x}}$$

$$=\frac{2\sin x\cos x}{\cos^2 x-\sin^2 x}$$

$$=\frac{\sin 2x}{\cos 2x}$$

p)

$$\frac{\sec x}{1+\sec x}=\frac{1-\cos x}{\sin^2 x}$$

$$\frac{\dfrac{\sec x}{\sec x}}{\dfrac{1}{\sec x}+\dfrac{\sec x}{\sec x}}=\frac{1-\cos x}{1-\cos^2 x}$$

$$\frac{1}{1+\cos x}=\frac{1-\cos x}{(1-\cos x)(1+\cos x)}$$

$$=\frac{1}{1+\cos x}$$